Exercise on duality and slutsky

Given the following indirect utility function $v = \dfrac{m^4}{16p_1^2p_2^2}$

1. Obtain the Marshallian demand for good x_1 and x_2 and interpret its degree of homogeneity in prices and income.
2. If $m = 16$, $p_1 = 1$ and $p_2 = 2$, and then the price of good 1 changes to 4, what percentage of the change in the quantity demanded for this good is due to the income effect?

Solutions

1. We use Roy's identity:

$$\frac{\frac{\partial v}{\partial p_1}}{\frac{\partial v}{\partial m}} = -x_1^m = \frac{\frac{-2m^4}{16p_1^3p_2^2}}{\frac{4m^3}{16p_1^2p_2^2}} = \frac{-2m^4 16p_1^2p_2^2}{16p_1^3p_2^2 4m^3} = -\frac{m}{2p_1}$$

$$\frac{\frac{\partial v}{\partial p_2}}{\frac{\partial v}{\partial m}} = -x_2^m = \frac{\frac{-2m^4}{16p_1^2p_2^3}}{\frac{4m^3}{16p_1^2p_2^2}} = \frac{-2m^4 16p_1^2p_2^2}{16p_1^2p_2^3 4m^3} = -\frac{m}{2p_2}$$

We find the degree of homogeneity of x_1^m

$$x_1^m(\lambda p_1, \lambda p_2, \lambda m) = \frac{m\lambda}{2p_1\lambda} = \lambda^0 x_1^m$$

It has a degree of homogeneity 0, which means that an increase in prices and income in the same proportion does not generate a change in demand. Another way to understand it is that there is no money illusion.

2. If $m = 16$, $p_1 = 1$ and $p_2 = 2$ then:

$$x_1^m = \frac{16}{2 * 1} = 8$$

And if $p_1 = 4$:

$$x_1^{'m} \frac{16}{2 * 4} = 2$$

Also, the utility in each case would be:

$$v(16, 1, 2) = \frac{16^4}{161^2 2^4} = 1024$$

The reduction is $8 - 2 = 6$. Part of the demand reduction is due to the income effect and another part to the substitution effect. To calculate this, we need to see the change in Hicksian demand. First, we obtain the expenditure function:

$$(16up_1^2p_2^2)^{1/4} = E$$

By Shepard's lemma:

$$E'p_1 = x_1^h = \frac{1}{4}(16up_1^2p_2^2)^{-3/4} 32up_1p_2^2 = 8(16up_1^2p_2^2)^{-3/4} up_1p_2^2$$

We see the change in Hicksian demand, if $m = 16$, $p_1 = 1$ and $p_2 = 2$ then:

$$x_1^h = 8(16(1024)4)^{-3/4}(1024)4 = 32(1024)(56(1024))^{-3/4} = 8$$

$$x_1^{'h} = 8(16(1024)16 * 4)^{-3/4}(1024)4 * 4 = 4$$

So, the substitution effect is:

$$x_1^{'h} - x_1^h = 8 - 4 = 4$$

Therefore, now we can break down the total effect into income and substitution effects:

$$TE = IE + SE$$

$$6 = IE + 4$$

So the income effect is $6 - 4 = 2$

To find the percentage: $2/6 = 1/3 = 0.33$, that is, 33%